Math 185: The Art of Proof[1]

Duke University
Fall 2021

Instructor:	Stephen McKean	**Time:**	WF 8:30am – 9:45am
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"Guided only by their feeling for symmetry, simplicity, and generality, and an indefinable sense of the fitness of things, creative mathematicians now, as in the past, are inspired by the art of mathematics rather than by any prospect of ultimate usefulness." – Eric Temple Bell

Materials: The course textbook is *An Infinite Descent into Pure Mathematics* by Clive Newstead, freely available at https://infinitedescent.xyz. The course website can be found on Sakai.

Objectives: This course is designed to introduce mathematical proofs as a creative endeavor. The target audience of this course are students who would not normally be introduced to proofs in their major's curriculum. Successful students of this course can expect to achieve the following learning objectives:

- Improve problem solving skills, especially when facing a new and unfamiliar type of problem.
- Understand the role of abstraction in problem solving.
- Learn symbolic logic, as well as basic elements of set theory, number theory, and combinatorics.
- Practice typesetting mathematical and scientific writing using LaTeX.
- Practice presenting technical material in a clear way, with an emphasis on providing appropriate motivation and illuminating steps that require creativity.

Suggested Prerequisites: The only prerequisite for this course is a willingness to actively participate in class and engage with difficult material. This course is primarily intended for students whose majors do not include extensive math requirements. Students majoring in the arts, humanities, life sciences, and social sciences are encouraged to register.

Classroom Climate: Despite being a *calculus for poets* style course, this class will not be an "easy version" of Math 245. This course will require hard work and will likely feel difficult during the first month of the semester. However, it is my goal as the instructor to make this course welcoming and rewarding for all. I will actively work to create an environment in which students feel excited to ask questions, offer suggestions, and make mistakes. I ask that all students also commit to making this course a supportive and kind community.

Academic Integrity: All students are expected to be honest with themselves, their peers, and the instructor. Students are encouraged to collaborate with each other on assignments. Students are **strongly discouraged** from consulting outside references, as this can severely impede the learning process. All resources utilized in completing an assignment, including collaborators, instructor help, and outside resources, should be explicitly credited for their help.

Accommodations and Questions: Please reach out to me in person or via email if you have any questions, need any additional support, or if you would like to discuss expectations, accommodations, or concerns.

Course Structure: The course will consist of in-person interactive lectures, weekly homework sets, monthly projects, and a final project. There will be grades associated to each of these course components. (If circumstances prevent in-person coursework, asynchronous virtual mini-lectures and synchronous discussion sessions will replace the in-person interactive lectures.)

[1] This syllabus is adapted from Emily Riehl's Math 301 syllabus at Johns Hopkins University.

- **Lectures (5% attendance, 10% participation):** Students are expected to attend lectures. Every lecture will consist of many breaks for exercises, questions, and discussions. Attendance and participation credit will be recorded during each lecture. Every student is allowed 2 unexcused absences. Any other absences should be discussed with the instructor as soon as possible.
- **Weekly homework (50%):** Each week, there will be a short homework assignment to be completed and typed up using LaTeX. Students are encouraged to collaborate on these assignments. The lowest homework grade will be dropped.
- **Monthly projects (30%):** There will be three monthly projects (September, October, November), each worth 10% of the final grade. The goal of the monthly projects is to have students learn material well enough that they can teach the material. Each monthly project will consist of three parts:
  - (i) Choosing a theorem to focus on for the month (2% out of 10%). Students should consult with the instructor to choose a suitable theorem to learn about. These choices will be due by the end of the first full school week of the month. No credit will be given for completing this part of the project late, but students can still receive full credit for the remaining two parts.
  - (ii) Presenting the proof in class (4% out of 10%). Students will learn the relevant definitions and lemmas needed to prove their chosen theorem. Students will then present this material at the board and receive feedback from their peers. Presentations may vary in length but will probably be longer than 5 minutes. Presentations must be given during the calendar month of the project. Students are responsible for scheduling a presentation time with the instructor.
  - (iii) Writing a report (4% out of 10%). Due by the end of the month, students should prepare a report detailing the definitions, lemmas, and proof necessary to explain their chosen theorem. There is no minimum length for this report, but an appropriate topic will probably take at least 2 pages to write up. This report should be typeset with LaTeX.
- **Final project (5%):** Students will prepare a written report of their favorite topics from the course. This report should include a technical discussion of the results, as well as an explanation about the student's interest in the results.

Grading Policy: Letter grades will be assigned as follows:

A+: $[97\%, 100\%]$, **A:** $[93\%, 97\%)$, **A-:** $[90\%, 93\%)$
B+: $[87\%, 90\%)$, **B:** $[83\%, 87\%)$, **B-:** $[80\%, 83\%)$
C+: $[77\%, 80\%)$, **C:** $[73\%, 77\%)$, **C-:** $[70\%, 73\%)$
D+: $[67\%, 70\%)$, **D:** $[63\%, 67\%)$, **D-:** $[60\%, 63\%)$

Tentative Course Outline: This is a potentially optimistic outline of the material we will cover.

Week 1: Overview, motivation, and goals (§0)
Week 2: Propositional logic (§1.1), division (§6.1)
Week 3: Proof-writing (§1.1, Appendix A), prime numbers (§6.2)
Week 4: Variables and quantifiers (§1.2)
Week 5: Logical equivalence (§1.3), modular arithmetic (§6.3)
Week 6: Tautologies and proof strategies (§1.3)
Week 7: Sets and operations (§2.1), finite sets (§7.1)
Week 8: Functions, injections, and surjections (§2.2–2.3)
Week 9: Peano's axioms (§3.1)
Week 10: Weak induction (§3.2)
Week 11: Strong induction (§3.3)
Week 12: Counting principles (§6.2)
Week 13: Countable and uncountable sets (§9.1)
Week 14: Thanksgiving break (no class)
Week 15: Cardinality and cardinal arithmetic (§9.2–9.3)