Math 293X: Topological Modular Forms

Harvard University
Fall 2023

Instructor:	Stephen McKean	**Time:**	TH 9:00am – 10:15am
Email:	smckean@math.harvard.edu	**Place:**	SC B10

Website: Officially, the course website will be on Canvas. However, all course materials other than announcements will also be available at https://shmckean.github.io/teaching/tmf.

Description: This course is a case study in the interconnectivity of math. We will begin with Kepler's laws of planetary motion, pass through the Atiyah–Singer index theorem and elliptic cohomology, and end at the spectrum of topological modular forms. This is a beautiful story involving a wide range of topics including operator theory, modular forms, elliptic curves, string theory, and homotopy theory. The real goal of this course is to encounter and learn about various branches of math in an organic manner, with the historical development of topological modular forms serving as our leitfaden.

Suggested Prerequisites: There are no hard prerequisites for this course. I will try to aim the material at students who have taken first-year graduate courses in algebra, topology, and analysis. If you are interested in taking the course but are uncertain about your background knowledge, please reach out to me.

Topics: Here is a short list of some topics that I hope to cover in the course.

- Geometry/topology: cobordism, genera, index theory, spin and string structures.

- Algebra/arithmetic: elliptic curves, modular forms, formal group laws, (derived) stacks.

- Homotopy theory: generalized cohomology theories, spectra, E_∞-rings, spectral sequences.

Because of the broad range of subjects, we will not get to dive too deeply into any one topic. The goal is to learn how to use tools and ideas from different areas of math in concert. See the next page for an optimistic, tentative schedule of the course.

References: Course notes and external references will be compiled throughout the semester. Notes and references will be available on the course website.

Grading: I will assign one or two small problems to think about after each lecture. There is no need to submit solutions, since the real purpose of these problems is to help students stay engaged with the course material. However, if you need a grade for this course (for example, if you are an undergraduate), you should carefully write up solutions to 5 of these problems and submit them before Thanksgiving break. Feel free to discuss these problems with me or others as you prepare to submit them (but see the Academic Integrity section below). Alternatively, you can write a report on a topic related to the course. Please contact me if you are interested in writing a report.

Classroom Climate: I hope that this class will have participants from a broad range of mathematical fields. It is my goal as the instructor to make this course welcoming and rewarding for all. I will actively work to create an environment in which students feel excited to ask questions, offer suggestions, and make mistakes. I ask that all students also commit to making this course a supportive and kind community.

Academic Integrity: All students are expected to be honest with themselves, their peers, and the instructor. Students are encouraged to talk with each other about the daily problems. If you are submitting solutions or a report for a grade in the course, you should clearly acknowledge any people or resources that you consulted while preparing your work.

Accommodations and Questions: Please reach out to me in person or via email if you have any questions, need any additional support, or if you would like to discuss expectations, accommodations, or concerns.

Anticipated Schedule: This is an optimistic, tentative schedule of what I hope to cover in the course.

	Date	Topic	Date	Topic
Week 1	9/5	Intro and Kepler's laws	9/7	Elliptic functions
Week 2	9/12	Cobordism	9/14	Genera
Week 3	9/19	Index theory	9/21	Atiyah–Singer
Week 4	9/26	Spin geometry	9/28	Atiyah–Bott–Shapiro
Week 5	10/3	Elliptic genus	10/5	Elliptic curves and modular forms
Week 6	10/10	Witten genus	10/12	Spectra
Week 7	10/17	Formal group laws	10/19	Lubin–Tate theory
Week 8	10/24	Elliptic cohomology	10/26	Stacks
Week 9	10/31	Derived algebraic geometry	11/2	$\mathcal{M}_{1,1}$, $\overline{\mathcal{M}}_{1,1}$, and $\mathcal{M}_{\mathrm{fg}}$
Week 10	11/7	E_∞-ring spectra	11/9	TMF, Tmf, and tmf
Week 11	11/14	Theorem of the cube	11/16	String orientation
Week 12	11/21	Spectral sequences	11/23	Thanksgiving break
Week 13	11/28	$\pi_*\mathrm{tmf}$	11/30	$\pi_*\mathrm{tmf}$ (continued)
Week 14	12/5	TBD (buffer day)		